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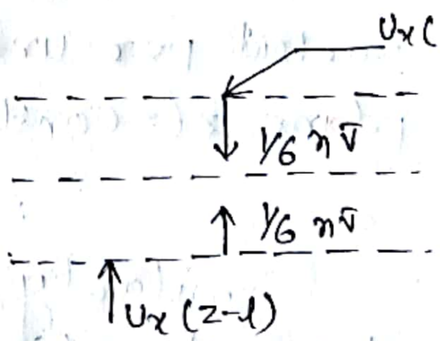
B.Sc (Part-1) Hons
Paper-11

Coefficient of viscosity.

Consider a gas in motion has a mean velocity components u_x in the x-direction. The magnitude of u_x depend on z so that we may write $u_x(z)$. Now consider any plane z = constant. As molecules cross back and forth across this plane, they carry x components of the momentum with them. Hence the gas below the plane z gains momentum in the x-direction because molecules coming from above the plane carry a larger x component of momentum with them. Conversely, the gas above the plane loses momentum in the x-direction because molecules ~~Comming~~ Coming from below the plane carry a smaller x component of momentum with them. Due to thermal motion, the number of molecules moving along the x-axis must be, on average the same as that moving along the y- or the z- axis.



Hence, if there are n molecules per unit volume one third of them may be considered as moving along the z-axis both up and down. Consider that c represents the mean molecular velocity corresponding to the temperature T. Then $\frac{1}{6}n$ molecules have the velocity c in the z direction and $\frac{1}{6}n$ molecules having the velocity in the z-direction since all those molecules contained in a cylinder of base of unit area and height c cross the plane z. Therefore are $\frac{1}{6}n\bar{c}$ malecules which cross a unit area of the plane per sec in the upward direction. Similarly $\frac{1}{6}n\bar{c}$ malecules cross a unit area of this plane from above.

Consider that every molecule in the gas, traverses the average distance l equal to the mean free path. It implies that molecules which cross the plane from below have, on the average experienced their preceding collisions at a distance l below the plane.

Since $u_x = u_x(z)$ is a function of z, the molecules lying in the plane $z = z-l$ had velocity $u_x(z-l)$. Thus each molecules of mass m transport a momentum $mu_x(z-l)$.

The mean value of momentum transported per unit time per unit area across the plane z(= Constant) in the upward direction.

$$= \frac{1}{6}n\bar{c}mu_x(z-l) \quad \text{——— (1)}$$

Similarly, consider malecules coming from above the plane where they suffered their preceding collision at $(z+l)$, we obtain the mean component of momentum transferred per unit time per unit area across the plane in the downward direction.

$$= \frac{1}{6}n\bar{c}mu_x(z+l) \quad \text{——— (2)}$$

On subtracting (1) from (2), we get the net gain in momentum

$$\frac{1}{6} n\bar{c}\{m u_x(z+l)\} - \frac{1}{6} n\bar{c}\{m u_x(z-l)\}$$

Carrying out Taylor's expansion

$$u_x(z+l) = u_x(z) + l\frac{\partial u_x}{\partial z}$$

$$u_x(z-l) = u_x(z) - l\frac{\partial u_x}{\partial z}$$

We get $F = \frac{1}{6} n\bar{c}\, m\, 2\frac{\partial u_x}{\partial z} l.$

On comparing $F = \eta\frac{\partial u_x}{\partial z}$, we get

$$\eta = \frac{1}{3} n\bar{c} m l = \frac{1}{3}\bar{c}\rho l \quad (\because \rho = mn)$$

η is independent pressure :- The mean free path is given by

$$l = \frac{1}{\sqrt{2}\pi n d^2}$$

Thus, we get $\eta = \frac{1}{3} n\bar{c} m \frac{1}{\sqrt{2}\pi n d^2} = \frac{1}{3\sqrt{2}} \cdot \frac{m\bar{c}}{\pi d^2}$

We can write the average speed $\bar{c}$, as

$\bar{c} = \sqrt{\frac{8KT}{m\pi}}$, Therefore, coefficient of viscosity η is given by

$$\eta = \frac{1}{3\sqrt{2}}\frac{m}{\pi d^2}\sqrt{\frac{8KT}{m\pi}}$$

$$= \frac{2}{3\pi d^2}\sqrt{\frac{mKT}{\pi}} = \frac{2}{3\pi d^2}\left(\frac{mKT}{\pi}\right)^{1/2} \quad \text{---(3)}$$

Hence η is independent of n. Thus at given temprature η is independent of gas pressure $p = nKT$

η varries as $\sqrt{T}$. For given gas, we can write

$$\eta = A\sqrt{T}$$

Where A is constant and $A = \frac{2}{3\pi d^2}\sqrt{\frac{mK}{\pi}}$

Hence η varries directly as square root of absolute temprature.

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